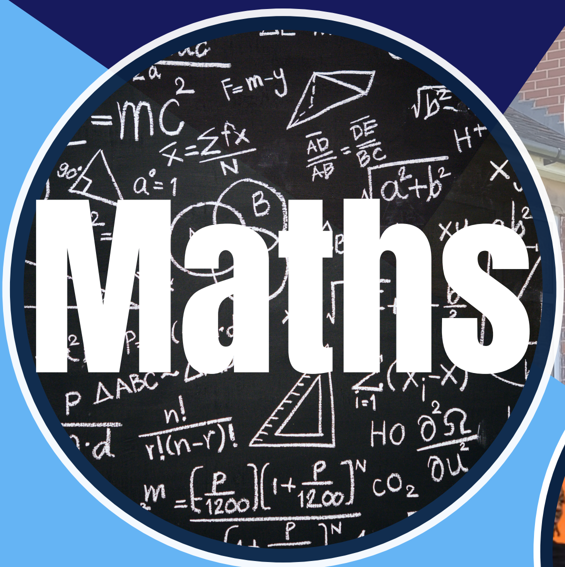
Saint Benedict
Catholic Voluntary Academy

2026

BE PREPARED

Maths



St Ralph Sherwin
Catholic Multi Academy Trust

To start an A-level in Mathematics you will need a secure knowledge of all aspects of Higher GCSE Mathematics. A fluency in the use of number and algebra is important, as is a good understanding of the fundamentals of geometry. These are detailed at the end of this document

The following pages are the type of knowledge that will be **immediately** useful to you as you start A-level and therefore an expectation as you start the course.

Algebra you should know.

- **You can use the laws of indices to simplify powers of the same base.**
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$
  - $(a^m)^n = a^{mn}$
  - $(ab)^n = a^n b^n$

To find the **product** of two expressions you **multiply** each term in one expression by each term in the other expression.

$$
\begin{aligned}
(x+5)(4x-2y+3) &= x(4x-2y+3) + 5(4x-2y+3) \\
&= 4x^2 - 2xy + 3x + 20x - 10y + 15 \\
&= 4x^2 - 2xy + 23x - 10y + 15
\end{aligned}
$$

$x\times$

$5\times$

Multiplying each of the 2 terms in the first expression by each of the 3 terms in the second expression gives 2 × 3 = 6 terms.

Simplify your answer by collecting like terms.

You can write expressions as a **product of their factors**.

- **Factorising is the opposite of expanding brackets.**

Expanding brackets →

$$
\begin{aligned}
4x(2x+y) &= 8x^2 + 4xy \\
(x+5)^3 &= x^3 + 15x^2 + 75x + 125 \\
(x+2y)(x-5y) &= x^2 - 3xy - 10y^2
\end{aligned}
$$

← Factorising

- **A quadratic expression has the form $ax^2 + bx + c$ where a, b and c are real numbers and $a \neq 0$.**

To factorise a quadratic expression:

- Find two factors of ac that add up to b — For the expression $2x^2 + 5x - 3$, $ac = -6 = -1 \times 6$ and $-1 + 6 = 5 = b$.
- Rewrite the b term as a sum of these two factors — $2x^2 - x + 6x - 3$
- Factorise each pair of terms — $= x(2x - 1) + 3(2x - 1)$
- Take out the common factor — $= (x + 3)(2x - 1)$

- **$x^2 - y^2 = (x + y)(x - y)$**

You can use the laws of indices with any rational power.

- $a^{\frac{1}{m}} = \sqrt[m]{a}$
- $a^{\frac{n}{m}} = \sqrt[m]{a^n}$
- $a^{-m} = \dfrac{1}{a^m}$
- $a^0 = 1$

If n is an integer that is **not** a square number, then any multiple of $\sqrt{n}$ is called a surd. Examples of surds are $\sqrt{2}$, $\sqrt{19}$ and $5\sqrt{2}$.

Surds are examples of **irrational numbers**.
The decimal expansion of a surd is never-ending and never repeats, for example $\sqrt{2} = 1.414213562\ldots$

You can use surds to write exact answers to calculations.

- **You can manipulate surds using these rules:**
  - $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
  - $\sqrt{\dfrac{a}{b}} = \dfrac{\sqrt{a}}{\sqrt{b}}$

If a fraction has a surd in the denominator, it is sometimes useful to **rearrange** it so that the denominator is a **rational** number. This is called rationalising the denominator.

- **The rules to rationalise denominators are:**
  - **For fractions in the form $\dfrac{1}{\sqrt{a}}$, multiply the numerator and denominator by $\sqrt{a}$.**
  - **For fractions in the form $\dfrac{1}{a + \sqrt{b}}$, multiply the numerator and denominator by $a - \sqrt{b}$.**
  - **For fractions in the form $\dfrac{1}{a - \sqrt{b}}$, multiply the numerator and denominator by $a + \sqrt{b}$.**

You should be able to answer these questions

1 Simplify:

a $y^3 \times y^5$ **b** $3x^2 \times 2x^5$ **c** $(4x^2)^3 \div 2x^5$ **d** $4b^2 \times 3b^3 \times b^4$

2 Expand and simplify if possible:

a $(x + 3)(x - 5)$ **b** $(2x - 7)(3x + 1)$ **c** $(2x + 5)(3x - y + 2)$

3 Expand and simplify if possible:

a $x(x + 4)(x - 1)$ **b** $(x + 2)(x - 3)(x + 7)$ **c** $(2x + 3)(x - 2)(3x - 1)$

4 Expand the brackets:

a $3(5y + 4)$ **b** $5x^2(3 - 5x + 2x^2)$ **c** $5x(2x + 3) - 2x(1 - 3x)$ **d** $3x^2(1 + 3x) - 2x(3x - 2)$

5 Factorise these expressions completely:

a $3x^2 + 4x$ **b** $4y^2 + 10y$ **c** $x^2 + xy + xy^2$ **d** $8xy^2 + 10x^2y$

6 Factorise:

a $x^2 + 3x + 2$ **b** $3x^2 + 6x$ **c** $x^2 - 2x - 35$ **d** $2x^2 - x - 3$

e $5x^2 - 13x - 6$ **f** $6 - 5x - x^2$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

7 Factorise:

a $2x^3 + 6x$ **b** $x^3 - 36x$ **c** $2x^3 + 7x^2 - 15x$

8 Simplify:

a $9x^3 \div 3x^{-3}$ **b** $(4^{\frac{3}{2}})^{\frac{1}{3}}$ **c** $3x^{-2} \times 2x^4$ **d** $3x^{\frac{1}{3}} \div 6x^{\frac{2}{3}}$

9 Evaluate:

a $\left(\frac{8}{27}\right)^{\frac{2}{3}}$ **b** $\left(\frac{225}{289}\right)^{\frac{3}{2}}$

10 Simplify:

a $\frac{3}{\sqrt{63}}$ **b** $\sqrt{20} + 2\sqrt{45} - \sqrt{80}$

11 **a** Find the value of $35x^2 + 2x - 48$ when $x = 25$.

b By factorising the expression, show that your answer to part **a** can be written as the product of two prime factors.

12 Expand and simplify if possible:

a $\sqrt{2}(3 + \sqrt{5})$ **b** $(2 - \sqrt{5})(5 + \sqrt{3})$ **c** $(6 - \sqrt{2})(4 - \sqrt{7})$

This question extends your knowledge

14 **a** Given that $x^3 - x^2 - 17x - 15 = (x + 3)(x^2 + bx + c)$, where b and c are constants, work out the values of b and c.

b Hence, fully factorise $x^3 - x^2 - 17x - 15$.

Solving Quadratics

A quadratic equation can be written in the form $ax^2 + bx + c = 0$, where a, b and c are real constants, and $a \neq 0$. Quadratic equations can have one, two, or no real solutions.

- **To solve a quadratic equation by factorising:**
  - **Write the equation in the form $ax^2 + bx + c = 0$**
  - **Factorise the left-hand side**
  - **Set each factor equal to zero and solve to find the value(s) of x**

Some equations cannot be easily factorised. You can also solve quadratic equations using the **quadratic formula**.

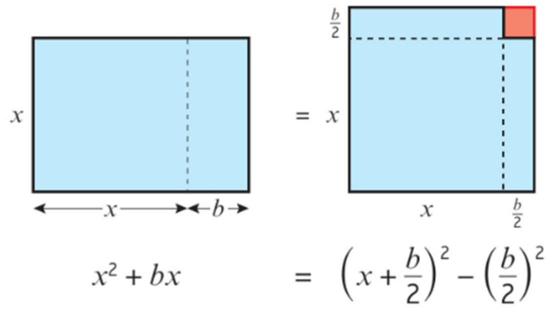
- **The solutions of the equation $ax^2 + bx + c = 0$ are given by the formula:**

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

It is frequently useful to rewrite quadratic expressions by **completing the square:**

- $x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$

You can draw a diagram of this process when x and b are positive:



The original rectangle has been rearranged into the shape of a square with a smaller square missing. The two areas shaded blue are the same.

- $ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

You should be able to answer the following questions

1 Solve the following equations without a calculator. Leave your answers in surd form where necessary.

a $y^2 + 3y + 2 = 0$ **b** $3x^2 + 13x - 10 = 0$ **c** $5x^2 - 10x = 4x + 3$ **d** $(2x - 5)^2 = 7$

4 Solve the following equations, giving your answers correct to 3 significant figures:

a $k^2 + 11k - 1 = 0$ **b** $2t^2 - 5t + 1 = 0$ **c** $10 - x - x^2 = 7$ **d** $(3x - 1)^2 = 3 - x^2$

5 Write each of these expressions in the form $p(x + q)^2 + r$, where p, q and r are constants to be found:

a $x^2 + 12x - 9$ **b** $5x^2 - 40x + 13$ **c** $8x - 2x^2$ **d** $3x^2 - (x + 1)^2$

Linear Simultaneous Equations

Linear simultaneous equations in two unknowns have **one set of values** that will make a pair of equations true at the same time.

The solution to this pair of simultaneous equations is $x = 5$, $y = 2$

$x + 3y = 11$ (1) —— $5 + 3(2) = 5 + 6 = 11$ ✓

$4x - 5y = 10$ (2) —— $4(5) - 5(2) = 20 - 10 = 10$ ✓

- **Linear simultaneous equations can be solved using elimination or substitution.**

You should be able to answer the following questions

1 Solve these simultaneous equations by elimination:

a $2x - y = 6$
$4x + 3y = 22$

b $7x + 3y = 16$
$2x + 9y = 29$

c $5x + 2y = 6$
$3x - 10y = 26$

d $2x - y = 12$
$6x + 2y = 21$

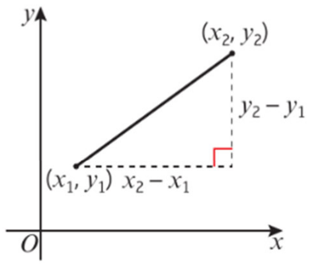
e $3x - 2y = -6$
$6x + 3y = 2$

f $3x + 8y = 33$
$6x = 3 + 5y$

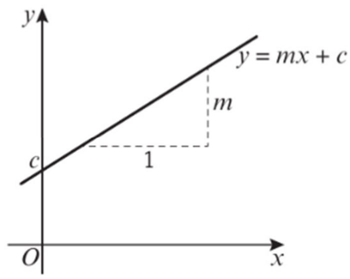
Straight Line Graphs

You can find the gradient of a straight line joining two points by considering the vertical distance and the horizontal distance between the points.

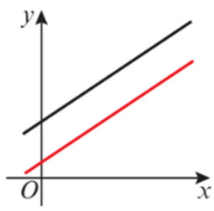
- **The gradient m of a line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula $m = \dfrac{y_2 - y_1}{x_2 - x_1}$**



- **The equation of a straight line can be written in the form $y = mx + c$, where m is the gradient and c is the y-intercept.**
- **The equation of a straight line can also be written in the form $ax + by + c = 0$, where a, b and c are integers.**

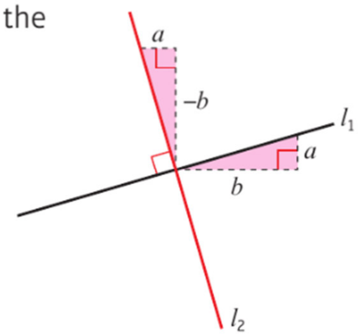


- **Parallel lines have the same gradient.**



Perpendicular lines are at right angles to each other. If you know the gradient of one line, you can find the gradient of the other.

- **If a line has a gradient of m, a line perpendicular to it has a gradient of $-\dfrac{1}{m}$**
- **If two lines are perpendicular, the product of their gradients is −1.**



The shaded triangles are congruent.

Line l_1 has gradient $\dfrac{a}{b} = m$

Line l_2 has gradient $\dfrac{-b}{a} = -\dfrac{1}{m}$

You should be able to answer the following questions.

1 Work out the gradients of the lines joining these pairs of points:

a $(4, 2), (6, 3)$
b $(-1, 3), (5, 4)$
c $(-4, 5), (1, 2)$
d $(2, -3), (6, 5)$
e $(-3, 4), (7, -6)$
f $(-12, 3), (-2, 8)$
g $(-2, -4), (10, 2)$
h $(\frac{1}{2}, 2), (\frac{3}{4}, 4)$
i $(\frac{1}{4}, \frac{1}{2}), (\frac{1}{2}, \frac{2}{3})$
j $(-2.4, 9.6), (0, 0)$
k $(1.3, -2.2), (8.8, -4.7)$
l $(0, 5a), (10a, 0)$

2 The line joining $(3, -5)$ to $(6, a)$ has a gradient 4. Work out the value of a.

1 Work out the gradients of these lines:

a $y = -2x + 5$
b $y = -x + 7$
c $y = 4 + 3x$
d $y = \frac{1}{3}x - 2$
e $y = -\frac{2}{3}x$
f $y = \frac{5}{4}x + \frac{2}{3}$
g $2x - 4y + 5 = 0$
h $10x - 5y + 1 = 0$
i $-x + 2y - 4 = 0$
j $-3x + 6y + 7 = 0$
k $4x + 2y - 9 = 0$
l $9x + 6y + 2 = 0$

2 These lines cut the y-axis at $(0, c)$. Work out the value of c in each case.

a $y = -x + 4$
b $y = 2x - 5$
c $y = \frac{1}{2}x - \frac{2}{3}$
d $y = -3x$
e $y = \frac{6}{7}x + \frac{7}{5}$
f $y = 2 - 7x$
g $3x - 4y + 8 = 0$
h $4x - 5y - 10 = 0$
i $-2x + y - 9 = 0$
j $7x + 4y + 12 = 0$
k $7x - 2y + 3 = 0$
l $-5x + 4y + 2 = 0$

1 Work out whether each pair of lines is parallel.

a $y = 5x - 2$
$15x - 3y + 9 = 0$

b $7x + 14y - 1 = 0$
$y = \frac{1}{2}x + 9$

c $4x - 3y - 8 = 0$
$3x - 4y - 8 = 0$

8 Find an equation of the line that passes through the point $(-2, 7)$ and is parallel to the line $y = 4x + 1$. Write your answer in the form $ax + by + c = 0$.

1 Work out whether these pairs of lines are parallel, perpendicular or neither:

a $y = 4x + 2$
$y = -\frac{1}{4}x - 7$

b $y = \frac{2}{3}x - 1$
$y = \frac{2}{3}x - 11$

c $y = \frac{1}{5}x + 9$
$y = 5x + 9$

d $y = -3x + 2$
$y = \frac{1}{3}x - 7$

e $y = \frac{3}{5}x + 4$
$y = -\frac{5}{3}x - 1$

f $y = \frac{5}{7}x$
$y = \frac{5}{7}x - 3$

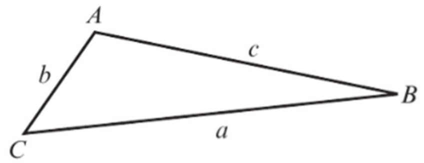
4 Find an equation of the line that passes through the point $(6, -2)$ and is perpendicular to the line $y = 3x + 5$.

Trigonometry

The cosine rule can be used to work out missing sides or angles in triangles.

- **This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:**

$$a^2 = b^2 + c^2 - 2bc \cos A$$



You can use the standard trigonometric ratios for right-angled triangles to prove the cosine rule:

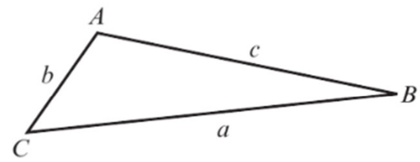
- **This version of the cosine rule is used to find an angle if you know all three sides:**

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

The sine rule can be used to work out missing sides or angles in triangles.

- **This version of the sine rule is used to find the length of a missing side:**

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



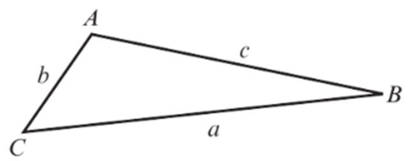
You can use the standard trigonometric ratios for right-angled triangles to prove the sine rule:

- **This version of the sine rule is used to find a missing angle:**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

You need to be able to use the formula for finding the area of any triangle when you know two sides and the angle between them.

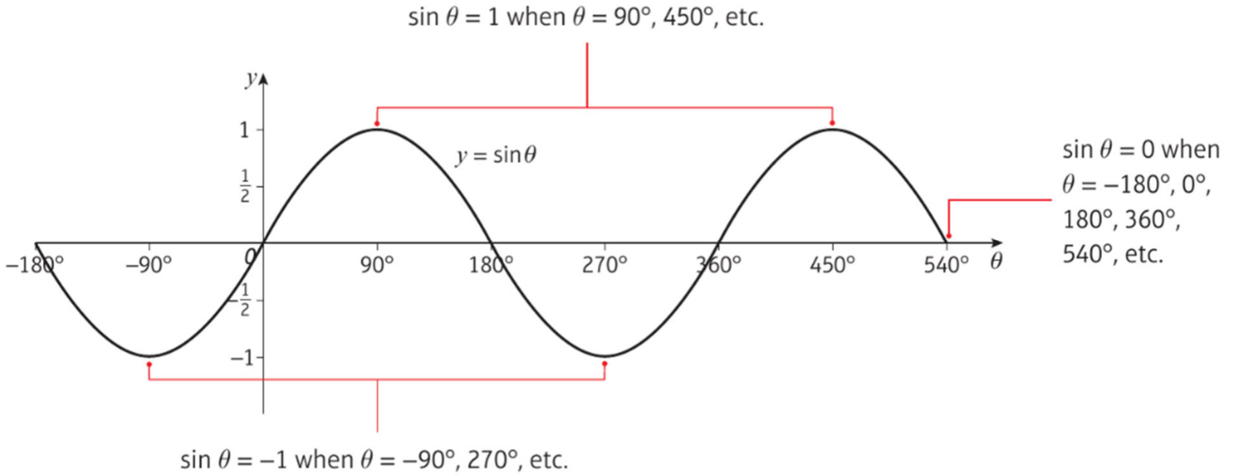
- **Area =** $\frac{1}{2}ab \sin C$



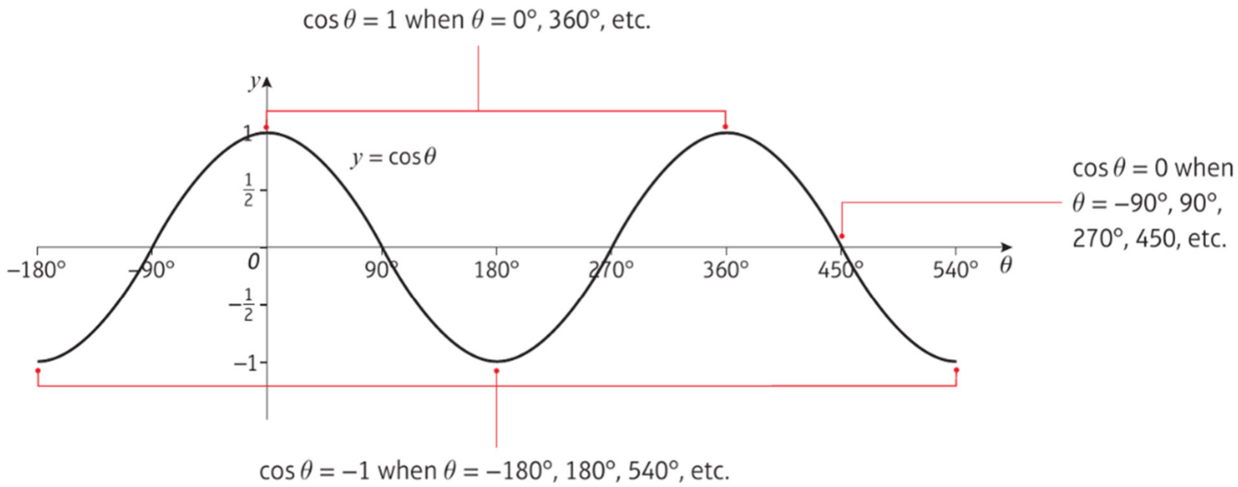
- **The graphs of sine, cosine and tangent are periodic. They repeat themselves after a certain interval.**

You need to be able to draw the graphs for a given range of angles.

- **The graph of $y = \sin\theta$:**
  - **repeats every 360° and crosses the x-axis at ..., −180°, 0, 180°, 360°, ...**
  - **has a maximum value of 1 and a minimum value of −1.**

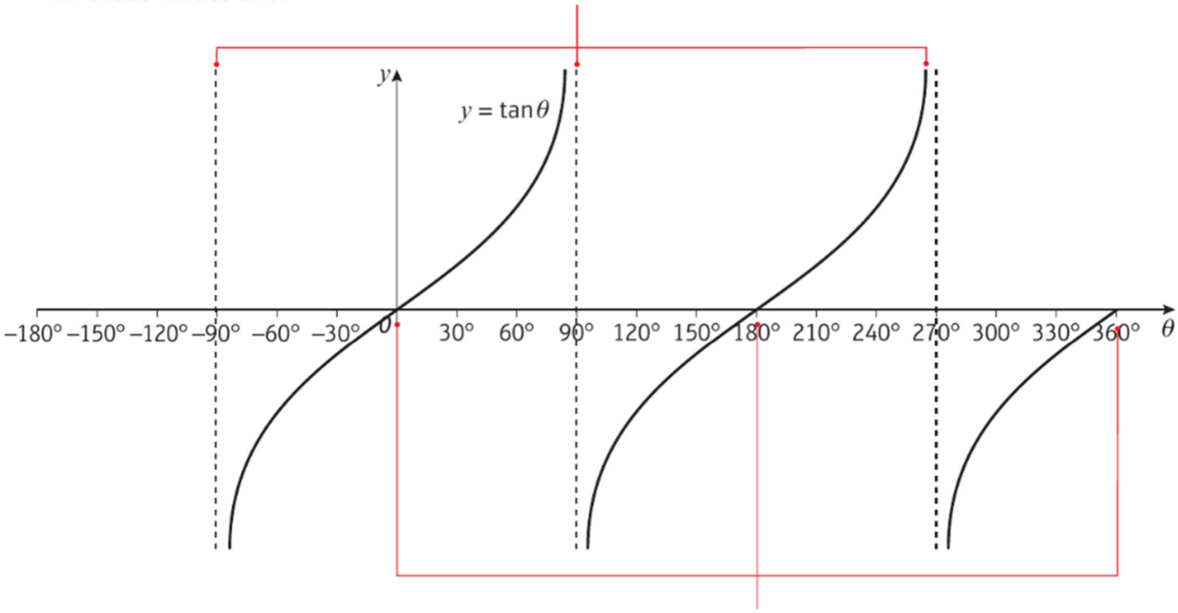


- **The graph of $y = \cos\theta$:**
  - **repeats every 360° and crosses the x-axis at ..., −90°, 90°, 270°, 450°, ...**
  - **has a maximum value of 1 and a minimum value of −1.**



- **The graph of $y = \tan\theta$:**
  - **repeats every 180° and crosses the x-axis at … –180°, 0°, 180°, 360°, …**
  - **has no maximum or minimum value**
  - **has vertical asymptotes at x = –90°, x = 90°, x = 270°, …**

$\tan\theta$ does **not** have maximum and minimum points but approaches negative or positive infinity as the curve approaches the **asymptotes** at –90°, 90°, 270°, etc. $\tan\theta$ is **undefined** for these values of θ.

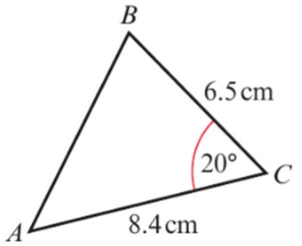


tan θ = 0 when θ = 0°, 180°, 360°, etc.

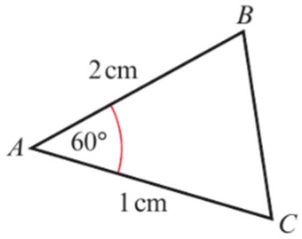
You should be able to answer the following questions.

1 In each of the following triangles calculate the length of the missing side.

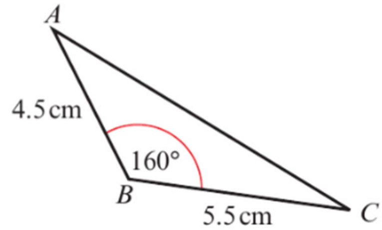
a



b



c



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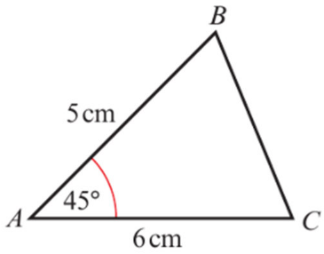

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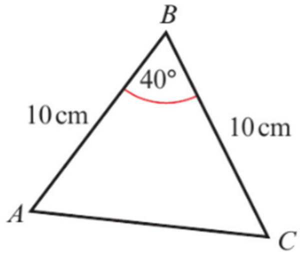

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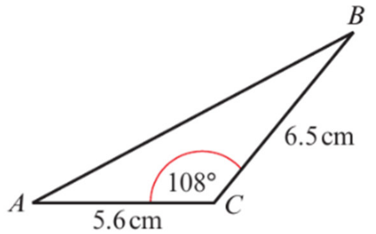
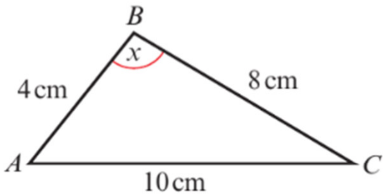
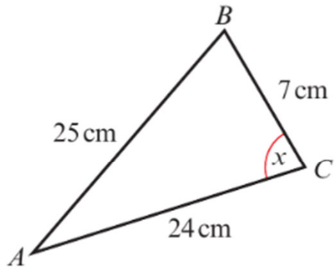


2 In the following triangles calculate the size of the angle marked x:

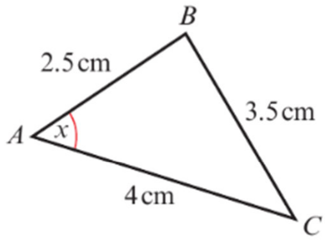
a



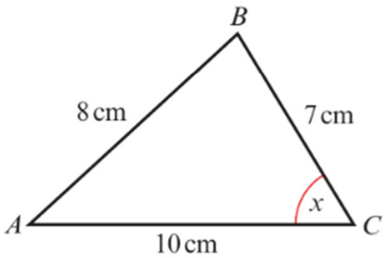
b



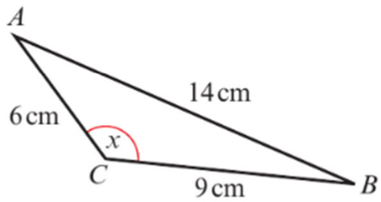
c



d



e



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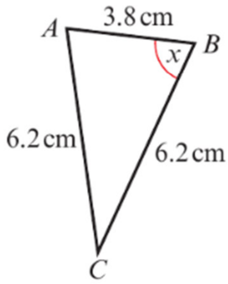
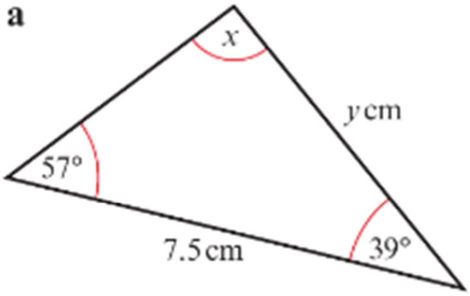
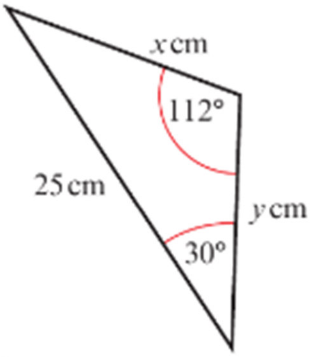


2 In each of the following triangles calculate the values of x and y.

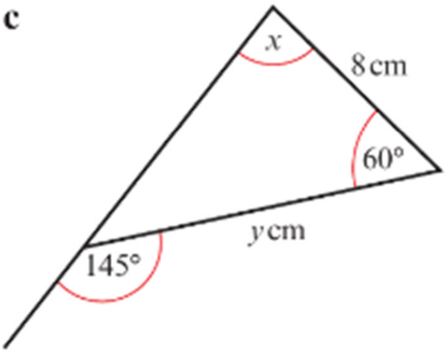
a



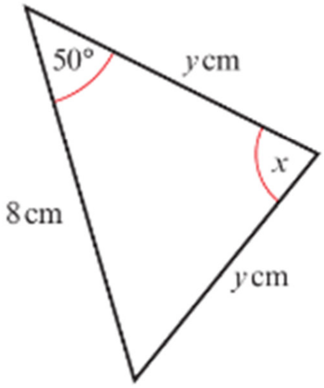
b



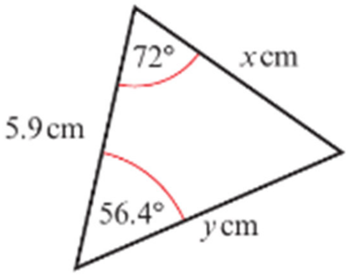
c



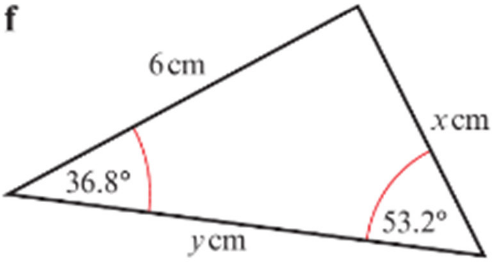
d



e

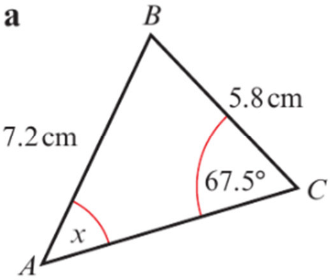


f

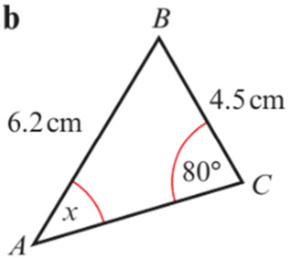


4 In each of the diagrams shown below, work out the size of angle x.

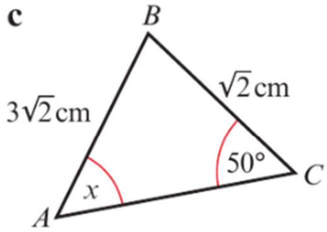
a



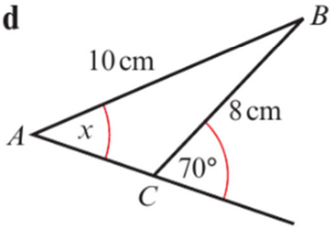
b



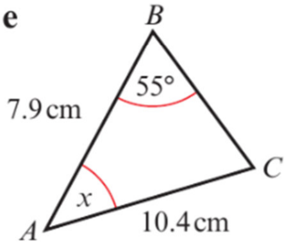
c



d

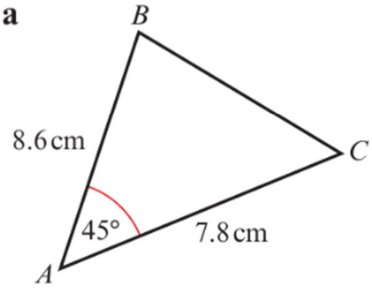


e



1 Calculate the area of each triangle.

a



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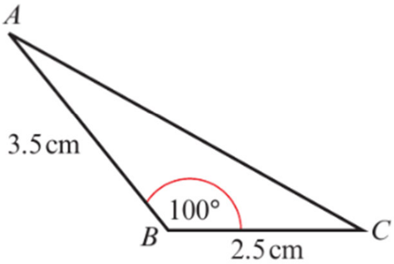

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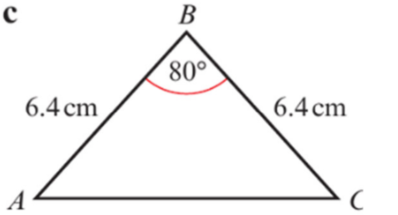
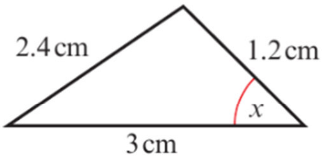
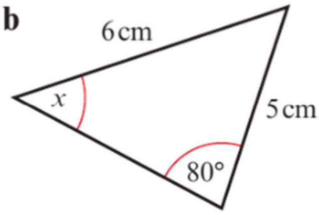


2 In each triangle below, find the size of x and the area of the triangle.

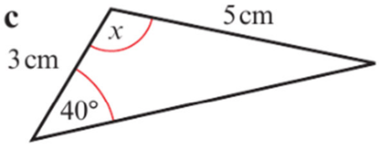
a



b

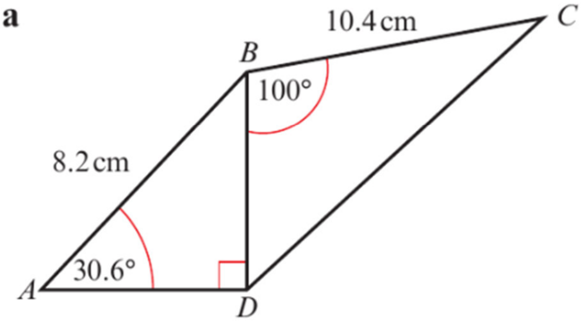


c



4 In each of the figures below calculate the total area.

a



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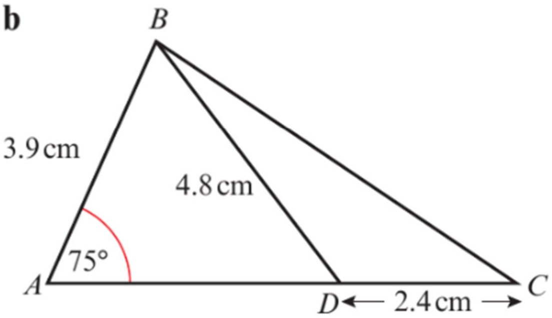

You should know these from GCSE Mathematics.

1. Rearranging formulae
2. Simplifying expressions
3. Factorising expressions
4. Solving Linear Equations
5. Functions
6. Vectors
7. Angle facts including circle theorems
8. Shape facts 2D and 3D
9. Statistical methods
10. Distance / speed / time calculations
11. Using and interpreting Graphs
12. Fraction calculations (algebra and number)

Course Overview. These are the topics that will be covered over the 2 years of the course.

Pure Maths (Two Thirds of the Course)

Algebraic Expressions

- Algebraic Methods
- Binomial Expansion
- Circles
- Differentiation
- Equations and Inequalities
- Exponentials and Logarithms
- Functions and Graphs
- Graphs and Transformations
- Integration
- Numerical Methods
- Quadratics
- Parametric Equations
- Radians
- Sequences and Series
- Straight Line Graphs
- Trigonometric Identities and Equations
- Trigonometric Functions
- Trigonometric Ratios
- Vectors

Mechanics (One sixth of the Course)

Constant Acceleration Formulae

Forces and Motion

Friction

Kinematics

Modelling

Moments

Projectiles

Variable Acceleration

Statistics (One sixth of the Course)

Correlation and Regression

Data Collection

Hypothesis Testing

Measures of Location and Spread

Normal Distribution

Probability

Representations of Data

Statistical Distributions

Final Exam

2 Papers on Pure

1 Paper on Applied (Mechanics and Statistics)